Stability of Monitoring Weak Changes in Multiply Scattering Media with Ambient Noise Correlation: Laboratory Experiments - Appendix

Céline Hadziioannou, Eric Larose, Olivier Coutant, Phillipe Roux and Michel Campillo

Laboratoire de Géophysique Interne et Tectonophysique, CNRS & Université J. Fourier, BP53, 38041 Grenoble, France. Email: eric.larose@ujf-grenoble.fr

APPENDIX A: CALCULATIONS

1. High Quality Data

In the stretching technique we are looking to maximize the cross correlation coefficient (equation (4) in the paper):

$$CC_k(\varepsilon) = \frac{\int_{t_1}^{t_2} h_k\left[t(1-\varepsilon)\right] h_0[t]dt}{\sqrt{\int_{t_1}^{t_2} h_k^2\left[t(1-\varepsilon)\right] dt \int_{t_1}^{t_2} h_0^2[t]dt}} \tag{A1}$$

In section IIIA we estimate CC_k for high quality data, without electronic or other noise. Our signals before and after a small temperature change then become:

$$h_0(t) = G_0(R,R,t) \otimes e(t) \tag{A2}$$

and

$$h_k(t) = G_k(R,R,t) \otimes e(t) = \left[G_0\left(R,R,t(1+\varepsilon_k)\right) + f(t)\right] \otimes e(t) \tag{A3}$$

where ε_k is the amount by which the record is stretched, and $f(t)$ represents the small fluctuations due to tiny physical changes in the medium as it expands slightly. Both h_0 and h_k are assumed to be stationary. Applying (A2) and (A3) to (A1), we get:

$$CC_k(\varepsilon) = \frac{\int_{t_1}^{t_2} \left[G_0\left(R,R,t(1+\varepsilon_k\right) + f(t)\right] \otimes e(t) \left[G_0(R,R,t)\right] \otimes e(t)dt}{\sqrt{\int_{t_1}^{t_2} \{\left[G_0\left(R,R,t(1+\varepsilon_k\right) + f(t)\right] \otimes e(t)\}^2 dt \int_{t_1}^{t_2} \{\left[G_0(R,R,t)\right] \otimes e(t)\}^2 dt}} \tag{A4}$$

We consider the simple case where $t_1 = 0$ and $t_2 = T$. We know that:

$$\rho(t) = \frac{e(t) \otimes e(t)}{\int e^2(t)} \tag{A5}$$

and simplify the expression to:

$$CC_k = \frac{\int_0^T \left[G_0^2 + G_0 f\right] \otimes \rho(t)\, dt}{\sqrt{\int_0^T \left[G_0^2\right] \otimes \rho(t)\, dt \int_0^T \left[G_0^2 + f^2 + 2G_0 f\right] \otimes \rho(t)\, dt}} \tag{A6}$$

Before calculating the mean value of CC_k, we assume that the Green functions at different times $G_0(t)$ and $G_0(t')$ are random, δ-correlated signals, with zero mean. This means that $\langle G(t)G(t')\rangle \approx \delta(t-t')$ and $\langle G_0\rangle = 0$. Furthermore, we suppose that the mean intensity of the Green function will remain unchanged before and after a temperature change: $\langle G_0^2\rangle = \langle G_k^2\rangle = \langle G^2\rangle$. The mean of any crossterms with the fluctuations $\langle G_0 f\rangle$ are set to zero. We use that $\langle \int_0^T G_0^2 dt\rangle = T\langle G_0^2\rangle$. With all this we can estimate the mean of CC_k:

$$A = \langle CC_k \rangle = \frac{T\langle G_0^2 \rangle}{T\sqrt{\langle G_0^2 \rangle (\langle G_0^2 \rangle + \langle f^2 \rangle)}} = \frac{\sqrt{\langle G_0^2 \rangle}}{\sqrt{\langle G_0^2 \rangle + \langle f^2 \rangle}} \tag{A7}$$

which is the constant A in equation (7) in the paper.

In order to find the amplitude of the fluctuations around the mean value we need to calculate the standard deviation of CC_k. We can first estimate its variance: $var(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2$. Breaking it up into smaller pieces, we start by calculating the mean of CC_k^2:

$$CC_k^2 = \frac{\int_0^T \left[G_0^2 + G_0 f\right] \otimes \rho(t)\, dt \int_0^T \left[G_0^2 + G_0 f\right] \otimes \rho(t')\, dt'}{\int_0^T [G_0^2] \otimes \rho(t)\, dt \int_0^T [G_0^2 + f^2 + 2G_0 f] \otimes \rho(t)\, dt} \tag{A8}$$

$$= \frac{\int_0^T \int_0^T \left[G_0^2 + G_0 f\right] \left[G_0^2 + G_0 f\right] \otimes \rho(t) \otimes \rho(t')\, dt dt'}{\int_0^T G_0^2 \otimes \rho(t)\, dt \int_0^T [G_0^2 + f^2 + 2G_0 f] \otimes \rho(t)\, dt} \tag{A9}$$

Again, crossterms with $\langle G_0 f \rangle$ are zero. The same assumptions as before equation (A7) hold, and we use that:

$$\int \rho(t)^2 dt \approx \frac{\Delta\omega}{2\pi} \tag{A10}$$

The mean value of CC_k^2 then becomes:

$$\langle CC_k^2 \rangle = \frac{2\pi}{\Delta\omega} \frac{T(\langle G_0^2 \rangle^2 + \langle G_0^2 \rangle \langle f^2 \rangle)}{T^2 \langle G_0^2 \rangle (\langle G_0^2 \rangle + \langle f^2 \rangle)}. \tag{A11}$$

Now the standard deviation is $\sqrt{var(CC_k)}$, or, using $\langle CC_k \rangle^2$ from equation (A7), $\sqrt{\langle B^2 \rangle} = \sqrt{\langle CC_k^2 \rangle - \langle CC_k \rangle^2}$:

$$\sqrt{\langle B^2 \rangle} = \sqrt{\frac{2\pi}{\Delta\omega\, T}} \frac{\sqrt{\langle f^2 \rangle}}{\sqrt{\langle G_0^2 \rangle + \langle f^2 \rangle}}, \tag{A12}$$

which is equation (8) in the paper.

2. Low Quality Data

In section IIIB we consider a signal with some noise added, electronic or otherwise:

$$S_0 = h_0 + n_0 \tag{A13}$$

$$S_k = h_k + n_k \tag{A14}$$

The mean value of CC_k will be a bit different for this case:

$$CC_k(\varepsilon) = \frac{\int_0^T [h_0 + n_0]\, [h_k + n_k] \otimes e(t) \otimes e(t)\, dt}{\sqrt{\int_0^T \left[(h_0 + n_0) \otimes e(t)\right]^2 dt \int_0^T \left[(h_k + n_k) \otimes e(t)\right]^2 dt}} \tag{A15}$$

$$= \frac{\int_0^T [h_0 h_k + h_0 n_k + h_k n_0 + n_0 n_k] \otimes \rho(t)\, dt}{\sqrt{\int_0^T [h_0^2 + n_0^2 + 2h_0 n_0] \otimes \rho(t)\, dt \int_0^T [h_k^2 + n_k^2 + 2h_k n_k] \otimes \rho(t)\, dt}} \tag{A16}$$

We assume that the mean of the crossterms involving noise (*eg.* $\langle h_i n_i \rangle$ and $\langle n_i n_j \rangle$) are zero. We also assume that the mean of the main signal h will stay the same after a temperature change: $\langle h_0^2 \rangle = \langle h_k^2 \rangle = \langle h^2 \rangle$. With this, the mean of CC_k is:

$$A = \langle CC_k \rangle = \frac{\langle h^2 \rangle}{\langle h^2 \rangle + \langle n^2 \rangle}, \tag{A17}$$

which is equation (10) in the paper.

As before, the variance of CC_k is given by $var(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2$:

$$CC_k^2 = \frac{\int_0^T \int_0^T \left[(h_0 + n_0)^2 (h_k + n_k)^2\right] \otimes \rho(t) \otimes \rho(t')\, dt' dt}{T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2 \int_0^T \rho(t)^2 dt} \tag{A18}$$

$$= \frac{\int_0^T \int_0^T \left[(h_0^2 + n_0^2 + h_0 n_0)(h_k^2 + n_k^2 + h_k n_k)\right] \otimes \rho(t) \otimes \rho(t')\, dt' dt}{T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2 \int_0^T \rho(t)^2 dt} \tag{A19}$$

Again, crossterms with noise are set to zero. Using equation (A10), the mean of CC_k^2 is now:

$$\langle CC_k^2 \rangle = \frac{2\pi\, T \left[\langle h^2 \rangle^2 + \langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle\right]}{\Delta\omega\, T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2} \tag{A20}$$

and the variance of CC_k, using equation (A17):

$$var(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2 = \frac{2\pi}{\Delta\omega\, T} \frac{\langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle}{(\langle h^2 \rangle + \langle n^2 \rangle)^2} \tag{A21}$$

Now the standard deviation is just the square root of equation (A21):

$$\sqrt{\langle B^2 \rangle} = \sqrt{\frac{2\pi}{\Delta\omega\, T}} \frac{\sqrt{\langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle}}{\langle h^2 \rangle + \langle n^2 \rangle}, \tag{A22}$$

which is equation (11) in the paper.